UNSTEADY LINEAR MOTION OF A THIXOTROPIC VISCOPLASTIC LIQUID BETWEEN INFINITE PARALLEL PLATES

S. E. Agaeva

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 7, No. 5, pp. 146-147, 1966

Numerous investigations have shown that clay and cement solutions are thixotropic viscoplastic liquids and do not conform to Newton's laws of friction. The deformation behavior of such liquids with anomalous viscosity is characterized by the Shvedov-Bingham formula. The mechanical properties of such systems are characterized by two parameters—the structural viscosity and the ultimate shear stress.

The characteristic feature of systems with anomalous viscosity is that they become liquid after mechanical mixing and become thick again when they are left undisturbed. This isothermally reversible conversion of a gel to a sol by mechanical mixing was named thixotropy by Freundlich.

Thixotropy is of great importance in the drilling of gas and oil wells, since it characterizes the ability of the solution to hold in suspension heavier particles of drilled-out rock when circulation is stopped.

A change in the ultimate shear stress and structural viscosity with time is a characteristic property of a thixotropic viscoplastic liquid.

In this note it is assumed for the sake of self-similarity of the solution that the structural viscosity of a viscoplastic liquid is a linear function of the time, i.e., $\eta = \alpha t$, and the ultimate shear stress $\tau_0 = \mathrm{const}$.

It is known that in the unsteady motion of a viscoplastic liquid there are viscoplastic and elastic flow regions, the boundaries of which are functions of time.

In the general case of unsteady motion of a viscoplastic liquid we have a boundary-value problem with a moving boundary. The inverse problem is considered: In the solution of inverse problems a law of variation is assigned to the moving boundaries and the velocity distribution corresponding to this law is determined.

We consider the linear unsteady motion of a thixotropic viscoplastic liquid between parallel plates. We assume that motion takes place in the direction of the z axis with velocity v between two plates placed at a distance of 2h from one another.

The equation of motion in this case has the form

$$\rho \frac{\partial v_z}{\partial t} = \eta \frac{\partial^2 v_z}{\partial x^2} + \frac{\Delta p}{l} \qquad (x_0 \leqslant x \leqslant h) \tag{1}$$

where x_0 is the size of the core.

The boundary conditions will be

$$v_z(h, t) = 0 \qquad \left(\frac{\partial v_z(x, t)}{\partial x}\right)_{x_0} = 0,$$

$$2lx_0\rho\left(\frac{\partial v_z(x, t)}{\partial t}\right)_{x_0} + 2l\tau_0 = 2x_0\Delta p. \tag{2}$$

Let the motion of the moving particles be put in the form

$$x_0 = \beta t, \quad h = \gamma t.$$

Here β is a constant to be determined; γ is some assigned quantity.

When $l^{-1}\Delta p = A_1/t$, it is easy to establish by using dimensional theory that the solution of the formulated problem is self-similar and has the form

$$v = Af(\xi), \qquad A = \sqrt{\tau_0/\rho}. \tag{3}$$

Substituting (3) into Eq. (1), we obtain

$$\frac{d^2 f}{d\xi^2} + \frac{\xi}{B}\frac{df}{d\xi} + \frac{Q}{B} = 0 \qquad (\xi_0 \leqslant \xi \leqslant \xi_*)$$

$$\left(B = \frac{\alpha}{\rho A^2}, \ Q = \frac{A_1}{\rho A}\right). \tag{4}$$

The boundary conditions take the form

$$f(\xi_*) = 0, \qquad \left.\frac{df}{d\xi}\right|_{\xi=\xi_0} = 0. \tag{5}$$

The solution of Eq. (3) with condition (5) has the form

$$f(\xi) = -\frac{Q}{B}\int_{\xi_*}^{\xi} \exp\frac{-\xi^2}{2B} F(\xi)\, d\xi, \quad F(\xi) = \int_{\xi_0}^{\xi} \exp\frac{\xi^2}{2B}\, d\xi. \tag{6}$$

For the velocity we obtain

$$V = -\frac{Q}{B}\int_{\xi^*}^{\xi} F(\xi) \exp\frac{-\xi^2}{2B}\, d\xi, \qquad V = \frac{v}{A}. \tag{7}$$

The friction stress is given by the formula

$$T = -\frac{Q}{B} F(\xi) \exp\frac{-\xi^2}{2B} + T_0,$$

$$\left(T = \frac{\tau}{\alpha}, \quad T_0 = \frac{\tau_0}{\alpha}\right). \tag{8}$$

The self-similar solution for this problem is of importance for verifying the correctness of various approximate methods. For instance, we assess the applicability of the approximate Slezkin-Targ method. In Eq. (1) and in condition (2) we replace

$$\frac{\partial v(x, t)}{\partial t} \text{ and } \left(\frac{\partial v(x, t)}{\partial t}\right)_{x_0}$$

by the mean value over the viscoplastic flow region, i.e.,

$$\varphi(t) = \frac{1}{h - x_0}\int_{x_0}^{h} \frac{\partial v_z}{\partial t}\, dx. \tag{9}$$

The solution of the approximate equation of motion with due regard to boundary conditions (2) will be

$$V = \frac{\tau_0}{\alpha}\xi\left(1 - \frac{A\xi}{2\beta}\right) + \frac{\tau_0\gamma}{\alpha A}\left(\frac{\gamma}{2\beta} - 1\right). \tag{10}$$

Here β is determined (in both solutions) from the condition of equilibrium of the core. For the friction stress we obtain

$$T = 2 - \xi\frac{A}{\beta}, \qquad T = \frac{\tau}{\tau_0}. \tag{11}$$

We cite the results of calculations of V for some values from formulas (7) and (9)

$\xi\ 10^2 =$	6	7	8	9	10	11	
$V\ 10^6 =$	2368	2263	2043	1706	1254	685	(7)
$V\ 10^6 =$	2401	2298	2079	1745	1290	727	(8)

The results of calculations from the two formulas agree very well with one another. This shows the practical applicability of the approximate method.

REFERENCES

1. A. Kh. Mirzadzhanzade, Hydrodynamics of Viscoplastic and Viscous Liquids in Oil Extraction [in Russian], Izd. Azerneftneshr, 1959.

2. G. T. Gasanov and A. Kh. Mirzadzhanzade, "Solution of inverse problems of unsteady motion of a viscoplastic liquid," PMTF, no. 5, 1962.

4 June 1966 Baku